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PBA-003-1013008

Seat No. ________

B. Sc. (Sem. III) (CBCS) Examination

November / December – 2018

Mathematics : Paper-03(A)

(Real Analysis)

(New Course)

Faculty Code : 003

Subject Code : 1013008

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.

(2) Number given in the right indicate mark of question.

1 (A) Answer the following question briefly : **4**

(1) Define convergence of sequence.

(2) Discuss convergence of $\left\{\sin\left(\frac{n\pi}{2}\right)\right\}$.

(3) Discuss convergence of $\left\{\frac{n^2+3n+5}{2n^2+5n+7}\right\}$.

(4) Define monotonic sequence.

(B) Attempt any **one** out of **two** : **2**

(1) Prove that $\lim\limits_{n\to\infty}\frac{1}{n}(1+2^{1/2}+3^{1/3}+\cdots\cdots+n^{1/n})=1$.

(2) Every convergent sequence is bounded.

(C) Attempt any **one** out of **two** : **3**

(1) If $\lim_{n\to\infty} a_n = l$ then $\lim_{n\to\infty} \frac{a_1 + a_2 + \ldots\ldots + a_n}{n} = l$.

(2) Discuss the convergence of the seq.

$S_1 = \sqrt{2},\ \ S_{n+1} = \sqrt{2 + S_n}$.

(D) Attempt any **one** out of **two** : **5**

(1) Prove $\lim_{n\to\infty} \sqrt[n]{a} = 1,\ \ a > 0$.

(2) Prove the sequence $\{S_n\}$ defined by

$S_1 = \sqrt{7},\ \ S_{n+1} = \sqrt{7 + S_n}$ converges to the positive root of equation $x^2 - x - 7 = 0$.

2 (A) Answer the following questions briefly : **4**

(1) State P test.

(2) Define Power series.

(3) Define alternating series.

(4) State D' Alemberts Ratio test.

(B) Attempt any **one** out of **two** : **2**

(1) Test the convergence of the series

$$\frac{1}{1\cdot 2\cdot 3} + \frac{3}{2\cdot 3\cdot 4} + \frac{5}{3\cdot 4\cdot 5} + \frac{7}{4\cdot 5\cdot 6} + \ldots\ldots.$$

(2) Prove the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \ldots\ldots$ is convergent.

(C) Attempt any one out of **two** : **3**

(1) Test the convergence of the series

$$\sum \frac{1}{n^2} \sin\left(\frac{1}{n}\right).$$

(2) Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{n^2}{3^n}$$

(D) Attempt any **one** out of **two** : **5**

(1) Find the radius of convergence and interval of convergence of series

$$x + 2x^2 + 3x^3 + 4x^4 + \ldots\ldots.$$

(2) Discuss the convergence of series

$$\frac{1}{2} + \frac{1 \cdot 3}{2 \cdot 4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \ldots\ldots$$

3 (A) Answer the following questions briefly : **4**

(1) Define divergence, solenoidal function.

(2) Define curl, irrotational function.

(3) Find unit normal at the surface $x^2 y + 2xz = 4$ at point $(2, -2, 3)$.

(4) Define ∇ operator, Laplace Eq^n.

(B) Attempt any **one** out of **two** : **2**

(1) Prove $\nabla r^n = nr^{n-2}\overline{r}$.

(2) $\overline{F} = (x^3, y^3, z^3)$ prove grad $(div\overline{F})$ = $6\overline{r}$.

(C) Attempt any **one** out of **two** : **3**

(1) Prove $\nabla^2(\log r) = \frac{1}{r^2}$.

(2) Prove $div(r^n \overline{r}) = (n+3)r^n$.

(D) Attempt any **one** out of **two** : **5**

(1) $\overline{f}, \overline{g}$ are vector functions

prove $div(\overline{f} \times \overline{g}) = \overline{g} \cdot curl\,\overline{f} - \overline{f}.cur1\,\overline{g}$.

(2) Prove $\frac{1}{r}$ satisfy Laplace Eq^n

show Curl (grad r^n) = $\overline{0}$.

4 (A) Answer the following questions briefly : **4**

(1) Find $\iint\limits_R (x^2 + 2y)\,dxdy,\ R = [0,1,0,2]$.

(2) Write the relation between Cartesian co-ordinate and Polar co-ordinate.

(3) Find $\iiint\limits_R xy\,dx\,dy\,dz,$ Risq qube $0 \le x, y, z \le 1$.

(4) Write the area of region R.

(B) Attempt any **one** out of **two** : **2**

(1) Define double integral.

(2) Prove $\iint_R e^{x^2+y^2}\,dxdy = \pi(e-1)$,

where R is a circle $x^2 + y^2 \leq 1$.

(C) Attempt any **one** out of **two** : **3**

(1) Find $\int_0^a\int_0^b\int_0^c (x+y+z)\,dx\,dy\,dz$.

(2) Prove the volume of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

is $\frac{4}{3}\pi abc$.

(D) Attempt any **one** out of **two** : **5**

(1) Change the order of integral and evaluate

$$\int_0^9\int_{\sqrt{y}}^3 \sin(\pi x^3)\,dxdy,$$

(2) Prove $\iint_R (x+y)^3\,dxdy = \frac{255}{4}$

where R is bounded by lines $x+y=4,\ x+y=1,$

$x-2y=-2,\ x-2y=1$.

5 (A) Answer the following question briefly : **4**

(1) $B(1,1) =$ ________ .

(2) Find the value of $\boxed{\tfrac{1}{2}}$.

(3) State Stoke's theorem.

(4) Find $\int_{(0,0)}^{(2,2)} y^2 dx$.

(B) Attempt any **one** out of **two** : **2**

(1) Prove $\int_{(1,1)}^{(x,y)} 2xydx + (x^2 - y^2)dy$ is path independent find its value.

(2) Prove $\int_0^\infty \frac{x^8(1-x^6)dx}{(1+x)^{24}} = 0$.

(C) Attempt any **one** out of **two** : **3**

(1) Find $\iint_S x^3 dydz + x^2 ydzdx + x^2 zdxdy$ where S is a surface formed by $z = 0, z = b$ and a cylinder $x^2 + y^2 = a^2$.

(2) Prove $\int_a^b (x-a)^{m-1}(b-x)^{n-1}dx = (b-a)^{m+n-1}\beta(m,n)$.

(D) Attempt any **one** out of **two** : **5**

(1) State and prove Green's theorem for plane.

(2) State and prove the relation between Beta and Gamma function.
